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# **DATA FREQUENCY AND DEPENDENCE STRUCTURE IN STOCK MARKETS**

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# **DATA FREQUENCY AND DEPENDENCE STRUCTURE IN STOCK MARKETS**

## **ABSTRACT**

It has been shown that the univariate distributions and other properties of asset returns are sensitive to the data frequency but the effects of the data frequency on the dependence among returns have hardly been explored. We contribute to fill this gap by analysing the impact of frequency changes on the dependence structure across the returns of 100 highly-traded American stocks and the market return over the period 2000-2010. We show that, in some cases, the association between stock returns and the market return changes according to the data frequency and, in general, investments based on monthly trades tend to be more conservative than investments made on a daily basis.

JEL codes: G12, G11, C46

Keywords: data frequency; dependence structure; stock markets

## **1. INTRODUCTION**

There is ample evidence in the literature concerning the difference in some properties of univariate asset returns at diverse frequencies. Nonetheless, studies on the impact of the frequency variation on the dependence across returns are scarce. We aim to provide further insights into this issue by verifying the dependence structure between stock returns and the market return at two different frequencies (daily and monthly).

We analyse 100 American stocks in the period 2000-2010 and conclude that the data frequency affects the relationship between the return of each stock and the overall market return. The main practical implication of this study is that trading frequency has an influence on investment risk profile such that daily trades tend to yield more speculative results than monthly trades. That is, the probability of joint extreme events (losses or gains) in daily data was higher than in monthly data for most of the stocks considered.

## **2. DATA FREQUENCY, ASSET RETURNS AND DEPENDENCE**

### **2.1. Data frequency and properties of univariate asset returns**

A number of empirical studies have found that daily and monthly returns present different distributions and the latter tend to be closer to the normality (Cherubini et al., 2010, p. 35). Pedersen and Hwang (2002), for example, point out that monthly returns are aggregations of several daily returns and, according to the Central Limit Theorem, approach the normality. The data analysed by those authors confirms that returns becomes strongly non-normal at higher frequencies (which present fewer trades than lower frequencies). Diebold (1988), Koedijk et al. (1990) and Nekhili et al. (2002) corroborate that conclusion in the context of exchange rates whose return distributions tend to the normality as data frequency decreases.

Other aspects of asset returns have also been studied with respect to diverse frequencies. Zhou (1996), for example, examines the impact of the data frequency on the autocorrelation and on the volatility of returns in foreign exchange markets. The author finds that the autocorrelation and the volatility of returns reduce as the frequency decreases.

Beltratti and Morana (1999) focus on the volatility process (autoregressive models) of exchange rate returns and conclude that the volatility of high-frequency data tends to be stabler than the volatility of low-frequency data. Goodhart and O'Hara (1997) comment on additional points such as the impact of high-frequency databases on inter-relationships between markets and on studies concerning the efficiency of markets.

## **2.2. Dependence structure**

The dependence structure across variables can be expressed by means of copula functions. These functions link univariate distributions (regardless of their shapes) to joint distributions and can capture asymmetric relationships such as more intense connection among extreme values which cannot be identified by models based on normality assumptions (see, for instance, Nelsen, 2006).

The dependence strength is measured by copula parameters which can be estimated through several methods (see McNeil et al., 2005). Among the most popular approaches, the Canonical Maximum Likelihood (CML) has been found to be the most efficient (Durrleman et al., 2000). This technique consists of two steps where the dataset is first converted into uniform variables, and, in a second step, the copula parameters are estimated by maximizing a log-likelihood function that includes the uniform variables and the copula parameters (see, e.g., McNeil et al., 2005).

In terms of the selection of the best-fit copulas among some candidates, the most reliable results have been obtained via the Empirical Copula method (see Berg, 2009 and Genest et al., 2009).

### **2.3. Data frequency and dependence structure**

To our knowledge, Breymann et al. (2003) is the only study that directly investigates dependence structure at different frequencies. The authors analyse two exchange rates (USD/DEM and USD/JPY) at eight time horizons (from one hour to one day) and find out that the Student t copula was the best representation for all frequencies tested (compared to another four copula families: Clayton, Frank, Gaussian and Gumbel). The degrees of freedom increase with the frequency which means that higher frequencies are closer to the Gaussian (Normal) dependence since the higher the degrees of freedom are the closer the Student t copula is to the Gaussian copula. So, this finding related to the dependence structure is consonant with the conclusions for univariate distributions according to which higher frequencies tend to the normality.

## **3. EMPIRICAL ANALYSIS**

In this section, we check the dependence between the return of some selected American stocks and the market return at two frequencies: daily and monthly. The S&P (Standard & Poor's) 500 index is used as a proxy for the market and our sample is composed of the 100 American stocks that had the highest market capitalisations on December 31<sup>st</sup> 2010. The data refer to returns without dividends from January 3<sup>rd</sup> 2000 to December 31<sup>st</sup> 2010 and were downloaded from The Center for Research in Security Prices / Wharton Research Data Services (CRSP/WRDS).

Given that we are checking up on possible different relationship structures at diverse time horizons, we consider four copula families that represent four distinct dependence structures:

symmetric without tail dependence (Gaussian), symmetric with tail dependence (Student t), asymmetric with left tail dependence only (Clayton), and asymmetric with right tail dependence only (Gumbel).

The parameters of the candidate copulas and the best-fit copula to characterise the association between each stock and the market were estimated following the CML and the Empirical Copula methods, respectively (cited in Section 2.2).

Table 1 presents the number of the copula families that characterise the dependence between the returns of each stock and the market in four subperiods (2000-2001, 2002-2004, 2005-2007 and 2008-2010) and in the whole sample period (2000-2010). In Panel A (referent to daily returns), we see the predominance of the Student t copula (which corroborates the results of Breymann et al., 2003 for exchange rates). Conversely, in Panel B (monthly returns), although the Student t copula was typically the most frequent, the representation is not homogeneous given that the other three copulas considered are also representative for many of the stocks. Note that, in the subperiod 2008-2010, which includes the recent market crash, the dependence for all stocks at daily frequency is represented by a unique copula (Student t). So, we find evidence that the dependence between stock returns and the market return in higher-frequency data (daily, in our case) is generally denoted by the Student t copula and such dependence becomes more heterogeneous in lower-frequency data (monthly).

Table 2 exhibits the properties of the relationships expressed by the best-fit copulas with regard to the connection across extreme values (tail dependence). The comparison between Panels A (daily returns) and B (monthly returns) confirms our prior conclusion: the dependence is more heterogeneous for the lower-frequency data in all periods considered. Pertaining to the daily returns, most of the stocks present right tail dependence (indicating that high stocks returns are

strongly associated with high market returns) with or without left tail dependence. The analysis for the monthly returns reveals that, compared to Panel A, fewer stocks have right tail dependence and more stocks present only left tail dependence (i.e. low stock returns intensely linked to low market returns) or no tail dependence.

Next, we investigate the implications of the aforementioned differences in terms of the probability of extreme returns and losses. We estimate the joint probability for each stock separately and the market (S&P 500 index) at two extreme levels: 5% and 10%. Then, we added up those probabilities and divided them by 100 (the number of stocks in our sample) to calculate the average for each period. Table 3 displays the *average* probability of joint extreme returns (or losses) concerning each stock and the market at the specified levels. The second column, for example, gives the average probability that a stock return is one of its 10% worst (lowest) historical values at the same time that the market return is one of its 10% worst historical values. The third, the fourth and the fifth columns show the average probability related to the lowest 5%, highest 5% and highest 10% of the values, respectively.

By comparing Panels A (daily returns) and B (monthly returns), we see that the average probability of joint extreme events is higher for daily returns at all levels and periods investigated. For instance, in the complete sample period (2000-2010), the probability of a stock return being at its lowest 10% level when the market return falls to its lowest 10% level is 0.0437 for the daily frequency (see Panel A, second column, last row) and 0.0354 for the monthly frequency (see Panel B, second column, last row). In other words, this means that daily trades tend to be more speculative inasmuch as their returns, when compared to monthly returns, are more likely to follow the extreme movements of the market (either positive or negative). Therefore, when the market has excessive gains or losses, those investments are prone to have



excessive gains or losses, respectively, as well. This happened more intensely in the subperiod 2008-2010 that comprises the recent “credit crunch” (the highest values for daily returns in both tails). On the other hand, if investors trade less frequently (here, at the end of the months), they become relatively more protected (in contrast to daily investments) against high losses in the market but do not take great advantage of high positive general returns.

Since the numbers presented in Table 3 are averages, those results could be led by few stocks with skewed return distributions and high excess kurtosis. Table 4 confirms that the previous result is valid for most of the stocks in our sample; that is, most of the stocks present higher probability of joint extreme returns (or losses) with the market for daily data than for monthly data.

As an example of the calculation regarding each of the stocks analysed, Table 5 shows the probabilities of joint extreme events estimated for the 10 stocks with the highest market capitalisations on December 31<sup>st</sup> 2010 and the market overall return comprising the whole period from 2000 to 2010. Note that the values for the lowest and the highest returns at the respective levels (5% and 10%) are the same. That is, the joint probability for the lowest 10% (5%) of the returns is equal to the joint probability for the highest 10% (5%) of the returns. This fact reflects the symmetry of the Student t copula that represents the dependence for those 10 stocks over the years 2000-2010.

#### **4. CONCLUSIONS**

This study contributes to the literature by investigating the effect of the data frequency on the dependence structure between asset returns since the existing literature dealing with different frequencies has typically focused on univariate returns. Moreover, while most empirical

researches in this field are related to the foreign exchange market, we use data on the stock market.

We find evidence that investments in stocks for longer periods have more conservative profiles given that, on average, the probability of extreme results in such investments is reduced. This finding reveals the same tendency identified in univariate analyses given that lower-frequency data tend to the normality which entails lower probability of excessive losses or excessive returns.

Our study can be extended to other markets (both geographically speaking and with respect to distinct products), longer periods and other frequencies (especially shorter time horizons such as intra-day, tick-by-tick, data).

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Table 1 – Best-fit copulas to the relationship between stock returns  
and the market return (2000-2010)

Panel A: Daily returns				
Period	Copula families			
	Gaussian	Student t	Clayton	Gumbel
2000-2001	6	73	12	9
2002-2004	3	94	1	2
2005-2007	4	96	0	0
2008-2010	0	100	0	0
2000-2010	0	99	1	0
Panel B: Monthly returns				
Period	Copula families			
	Gaussian	Student t	Clayton	Gumbel
2000-2001	22	36	18	24
2002-2004	13	43	24	20
2005-2007	26	24	35	15
2008-2010	17	42	39	2
2000-2010	11	68	13	8

This table displays the number of stocks represented by each copula family in the four subperiods considered and in the whole period (from 2000 to 2010). Total of stocks in each period: 100.

Table 2 - Dependence structure between stock returns and the market return (2000-2010)

Panel A: Daily returns					
Period	Tail dependence properties				
	No tail dependence	Left tail dependence only	Right tail dependence only	Left tail dependence	Right tail dependence
2000-2001	6	12	82	85	82
2002-2004	3	1	96	95	96
2005-2007	4	0	96	96	96
2008-2010	0	0	100	100	100
2000-2010	0	1	99	100	1
Panel B: Monthly returns					
Period	Tail dependence properties				
	No tail dependence	Left tail dependence only	Right tail dependence only	Left tail dependence	Right tail dependence
2000-2001	22	18	24	54	60
2002-2004	13	24	20	67	63
2005-2007	26	35	15	59	39
2008-2010	17	39	2	81	44
2000-2010	11	13	8	81	76

This table displays the number of stocks that presented the mentioned dependence structure in the four subperiods considered and in the whole period (from 2000 to 2010). The number of stocks listed in each row may be greater than the number of stocks analysed (100) given that some properties are not mutually exclusive.

Table 3 – Probability of joint extreme events between stock returns and the market return (2000-2010)

Panel A: Daily returns				
Period	Joint probability levels			
	Lowest 10% of the returns	Lowest 5% of the returns	Highest 5% of the returns	Highest 10% of the returns
2000-2001	0.0271	0.0108	0.0104	0.0263
2002-2004	0.0432	0.0191	0.0193	0.0436
2005-2007	0.0395	0.0167	0.0167	0.0395
2008-2010	0.0537	0.0252	0.0252	0.0537
2000-2010	0.0437	0.0199	0.0198	0.0435
Panel B: Monthly returns				
Period	Joint probability levels			
	Lowest 10% of the returns	Lowest 5% of the returns	Highest 5% of the returns	Highest 10% of the returns
2000-2001	0.0225	0.0084	0.0078	0.0213
2002-2004	0.0355	0.0151	0.0115	0.0294
2005-2007	0.0307	0.0126	0.0088	0.0241
2008-2010	0.0529	0.0244	0.0143	0.0366
2000-2010	0.0354	0.0149	0.0134	0.0327

The values in this table indicate the average probability of joint returns (concerning stocks and the market) in extreme scenarios. For example, the column “Lowest 10% of the returns” gives the probability that the worst (lowest) 10% of the observed stock returns happen at the same time as the worst 10% of the observed market returns. These probabilities were calculated for each of the 100 stocks analysed and the numbers in the table are the averages for each period.

Table 4 – Number of stocks for which the respective probability of joint extreme returns (losses) at the daily frequency is greater than at the monthly frequency

Period	Joint probability levels			
	Lowest 10% of the returns	Lowest 5% of the returns	Highest 5% of the returns	Highest 10% of the returns
2000-2001	69	70	66	70
2002-2004	72	73	88	89
2005-2007	74	75	90	89
2008-2010	50	53	88	85
2000-2010	86	86	93	91

Table 5 – Example of calculations of the probability of joint extreme events between the return of some stocks and the market return (2000-2010)

Panel A: Daily returns				
Stocks	Joint probability levels			
	Lowest 10% of the returns	Lowest 5% of the returns	Highest 5% of the returns	Highest 10% of the returns
Exxon	0.0457	0.0210	0.0210	0.0457
Apple	0.0427	0.0186	0.0186	0.0427
Microsoft	0.0416	0.0189	0.0189	0.0416
General Electric	0.0415	0.0189	0.0189	0.0415
Wal Mart	0.0404	0.0180	0.0180	0.0404
Chevron	0.0450	0.0208	0.0208	0.0450
IBM	0.0462	0.0205	0.0205	0.0462
Procter & Gamble	0.0425	0.0195	0.0195	0.0425
AT&T	0.0549	0.0262	0.0262	0.0549
Johnson & Johnson	0.0504	0.0238	0.0238	0.0504
Panel B: Monthly returns				
Stocks	Joint probability levels			
	Lowest 10% of the returns	Lowest 5% of the returns	Highest 5% of the returns	Highest 10% of the returns
Exxon	0.0287	0.0110	0.0110	0.0287
Apple	0.0384	0.0152	0.0152	0.0384
Microsoft	0.0410	0.0168	0.0168	0.0410
General Electric	0.0506	0.0244	0.0244	0.0506
Wal Mart	0.0259	0.0105	0.0105	0.0259
Chevron	0.0300	0.0110	0.0110	0.0300
IBM	0.0441	0.0181	0.0181	0.0441
Procter & Gamble	0.0187	0.0065	0.0065	0.0187
AT&T	0.0304	0.0112	0.0112	0.0304
Johnson & Johnson	0.0306	0.0135	0.0135	0.0306